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**Physics 101 Semester 1 2017**

**Formal Report No 2**

Understanding Fluid Dynamics by Applying the Continuity Equation and Bernoulli’s Equation 0to the Hole in a Beaker Experiment and a Venturi Tube.

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Abstract

Two experiments were carried out to use the continuity equation and Bernoulli’s equation in the hole in a beaker experiment and Venturi tube to improve understanding of fluid dynamics. The hole in a beaker experiment was conducted by filling a beaker timing how quickly it emptied. The rate of flow was used to calculate the velocity of water through the hole. The velocity of water through the hole was calculated to be (0.990 ± 0.273) ms-1 using the continuity equation. The velocity of the water was calculated to be (1.400 ± 0.007) ms-1 using Bernoulli’s equation. The difference in the velocity calculations was due to the continuity equation taking the water height and time into account whereas Bernoulli’s equation only took the height of the water into account. The Venturi tube was then used for experiment two by connecting a hose to the Venturi tube directly from the tap. The water flowed through the tube and the data was noted down. The velocity of water at each hole in the Venturi tube was calculated to be (0.368, 0.420, 0.458, 0.500, 0.554, 0.610 and 0.678) ms-1 using the continuity equation and (0.505, 0.423, 0.485, 0.542, 0.672, 0.729 and 0.863) ms-1 using Bernoulli’s equation. There was a percentage difference of (31, 0.7, 5.7, 8.1, 19, 18 and 24) % between the calculations using each equation. The varied percentage difference was due to the water pressure fluctuating between recording data. However, the equations used in these experiments were only theoretical as the water in the experiments could never have flowed with ideal flow. This was the major physical limitation in the experiments.

Introduction

Fluids can be found in any part of the world from the oceans surrounding land to the water cooling loops inside a desktop computer. The understanding of how fluids behave in different conditions is what allows researchers to use fluids in order to better the lives of people in the world by some shape or form. The manner in which fluids behave is known as fluid dynamics (The Audiopedia, 2016). Daniel Bernoulli discovered one of the most important relationships when dealing with fluid dynamics. He learned that just like a moving body or mass where kinetic energy and gravitational energy are conserved in an isolated system, fluids also have the same properties of conservation of energy (Hyper Physics, 2017). The general energy of a mass or body can be calculated using the equation:

|  |  |  |
| --- | --- | --- |
|  | = constant  (Physics Classroom, 2017) | (1) |

**M** is the mass of the body or object (kg), **v** is the velocity of the body or mass (ms-1), **g** is the gravitational constant (9.81ms-2) and **h** is the height of the body or mass (m). In this equation, the total energy of the mass or body is equal to the sum of the kinetic energy **(**and the gravitational potential energy **(**.

Bernoulli discovered fluids behaved in the same manner. He derived the following equation:

|  |  |  |
| --- | --- | --- |
|  | (PHYS101 S1 Lab Manual, 2017) | (2) |

In this equation, **P** is the pressure surrounding the fluid (pa), (rho)is the density of the fluid (kgm-3), **v** is the velocity of the fluid (ms-1), **g** is the gravitational constant and **h** is the height which the fluid is flowing. Therefore, in an isolated system, the energy of the fluid would not change as it would be converted from one form of energy to another.

The continuity equation

|  |  |  |
| --- | --- | --- |
|  | (Princeton University, 2017) | (3) |

demonstrates that the mass of a fluid in an isolated system is conserved when it is in motion (Princeton University, 2017) where **A1** and **A2** are the areas of the sections of pipes being compared and **V1** and **V2** are the velocities of the fluid at each respective part of the pipe. This equation shows that the velocity of a fluid decreases as the area of the pipe that it is contained in increases. This means that in the widest part of the pipe, the velocity of the fluid will be at its lowest.

Both these equations assume that the experiment is done with ideal flow of the fluid. This means that the fluid is incompressible, the flow is frictionless and it is laminar i.e. the fluid flows in parallel lines without creating any eddies, currents or whirlpools.

Combining these ideas means that the pressure in the pipe would be at its lowest wherever the thinnest part of the pipe is (PHYS101 S1 Lab Manual, 2017).

By replacing the pipe with a beaker, this idea can be proven visually and mathematically. The beaker must have a hole on the side of it closer to the bottom. The top of the beaker is the surface and the hole is considered the bottom of the beaker. If the two points being compared are at the same pressure, the pressure term is able to be removed from Bernoulli’s equation which was stated before. This results in the equation now being:

|  |  |  |
| --- | --- | --- |
|  | (PHYS101 S1 Lab Manual, 2017) | (4) |

This equation is especially useful when dealing with beakers that have a hole in them while they contain a fluid. This is because the equation describing the fluid at the top of the beaker should be the same as the equation used to describe the fluid flowing out of the hole in the beaker (the bottom) as both points are at the same pressure.

This creates the equation:

|  |  |  |
| --- | --- | --- |
|  |  | (5) |

Once again this can be simplified as because it can be assumed that the velocity at the surface of the fluid is zero as it empties. This is because the fluid is moving extremely slowly. The density of the liquid will be the same at the surface of the beaker and at the hole as the fluid does not change. This allows the densities () to cancel. In addition to this, the height at the bottom of the beaker is also zero as it is the end height in this system. Altogether, this reduces the equation to:

|  |  |  |
| --- | --- | --- |
|  |  | (6) |

This equation can be rearranged to find the velocity of the fluid through the hole. This makes the equation:

|  |  |  |
| --- | --- | --- |
|  |  | (7) |

Venturi Tube

A Venturi tube can also be used to demonstrate the use of both the continuity equation and Bernoulli’s equation. The apparatus is a tube that has a decreasing cross section and a tube is inserted into the fluid it contains. This allows for the pressure difference to be measured between the flowing fluid and the fluid which is stopped by the tube. Bernoulli’s equation is the best equation to use as it takes into account these different pressures.

|  |  |  |
| --- | --- | --- |
|  |  | (8) |

Since the density (rho) and gravitational constant are on both side of the equation, the equation can be divided though by the density and gravitational constant. Also, the velocity at the Pitot tube is zero since no water is flowing. Therefore v2 is zero. This creates the equation:

|  |  |  |
| --- | --- | --- |
|  |  | (9) |

This equation can be simplified by cancelling out the rho. Since there is no change in height of the apparatus, h1 and h2 are zero. This gives:

|  |  |  |
| --- | --- | --- |
|  |  | (10) |

Both these pressures can be calculated using the pressure height relationship where

|  |  |  |
| --- | --- | --- |
|  |  | (11) |

This can be rearranged to be in terms of h and then it can be substituted into the equation [number] to give:

|  |  |  |
| --- | --- | --- |
|  |  | (12) |

where h0static is the height of the water in the manometer tube for each hole and hdynamic is the height of the water in the Pitot tube.

This can finally be rearranged to find the velocity of the water at each hole. The equation becomes:

|  |  |  |
| --- | --- | --- |
|  |  | (13) |
|  |  |  |

The aim of this experiment was to use the continuity equation and Bernoulli’s equation in the hole in a beaker experiment and Venturi tube to improve understanding of fluid dynamics.

Method

Experiment One

The equipment required to carry out experiment one was set up as shown in **Figure 1** below. The inside diameter of both the beaker and the hole inside the beaker were measured with a Vernier Calliper. The beaker was then filled with water whilst the hole was closed with a group member’s finger. Once filled to the desired level, the beaker was placed on top of the workbench with the member’s finger still blocking the hole.

Hole

Height (scale was marked on the outside of the beaker).

Diameter (hole)

Beaker

Beaker to catch water

Diameter (beaker)

Figure 1: Experiment 1 setup. The beaker was filled to a height of 0.1m with a group member's finger covering the hole.

The stopwatch was started as soon as the member’s finger was removed from the hole. The water was allowed to drain out from the beaker into another other beaker which was held just below the hole of the beaker by another group member. The time required to fully drain the beaker with a hole was recorded in order to find the flow rate of the water leaving the beaker through the hole.

Experiment Two

The equipment used to complete experiment two was set up as can be seen in **Figure 2**. The Venturi apparatus was connected directly to the outlet of the sink with a hose pipe. The tap was slowly opened. This was important as it decreased the chances of creating any air bubbles. The flow rate was constantly monitored. If any air bubbles were present, they were removed by inserting a piece of wire into the respective manometer tube.

Manometer tube

hdynamic

(Tube 8)

Hstatic

Water flow in

Pitot tube

n = 7

n = 1

Figure 2: Setup for experiment 2. The water flow into the Venturi apparatus came directly from the tap.

First, the Pitot tube was pushed until the tip was just underneath the first manometer tube. The diameter of the tube at this point was measured with a Vernier Calliper. Then the height of the water inside the manometer tube was recorded once there were no major fluctuations in the height i.e. the system was stable. This was the hstatic as can be seen in the results table later in this report. The height of the water in the outer tube (pitot) was measured as well. This was noted as hdynamic in the results table. This process including measuring the diameter of the tube was repeated at each tube. There were seven tubes in total. Finally, the outlet of the Venturi apparatus was funnelled into a beaker. The beaker was filled to five hundred millilitres and the time it took was recoded. This was repeated three times in order to find the average flow rate of the apparatus.

Results

Measurements required for experiment one

Diameter of the beaker, x = (0.099 ± 0.001) m

Diameter of the hole, x = (0.006 ± 0.001) m

These uncertainties were chosen due to the calliper using one millimetre intervals.

The uncertainty for time was calculated by one person trying to make the timer read exactly one second. This was done five times. The timings they obtained were recorded and the uncertainty was half the range of the times.

|  |
| --- |
| Time Obtained (s) |
| 1.23  1.16  1.09  1.09  1.08 |

Table 1: Measured time (s) trying to achieve exactly one second.

The range of the times was 1.23 – 1.08 = 0.15 (s). Half the range = 0.075 (s). Therefore the uncertainty for time in the first experiment was ± 0.08 (s) as the stopwatch only read to two decimal places.

|  |  |
| --- | --- |
| Water Height ± 0.001 (m) | Time ± 0.08 (s) |
| 0.100  0.090  0.080  0.070  0.060  0.050  0.040  0.030  0.020  0.010  0.000 | 0.00  1.80  3.59  5.63  8.28  10.28  14.32  17.13  20.24  25.33  41.22 |

Table 2: Measured time (s) required to decrease water height (m).

Measurements required to carry out experiment two

All uncertainties in this experiment were ignored as instructed by the Laboratory Manual (PHYS101 S1 Lab Manual, 2017).

Hstatic was the height of the water above the measuring point in the nth tube (1-7). Hdynamic was the height of the water in tube number 8 at each interval.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Position (n) | Hstatic (m) | Hdynamic (m) | Diameter (m) | Area (m2) |
| 1  2  3  4  5  6  7 | 0.192  0.185  0.0.180  0.175  0.165  0.158  0.145 | 0.205  0.195  0.192  0.190  0.188  0.185  0.183 | 0.0145  0.0136  0.0130  0.0125  0.0119  0.0113  0.0107 | 1.65 × 10-4  1.45 × 10-4  1.33 × 10-4  1.22× 10-4  1.10 × 10-4  1.00 × 10-4  0.89 × 10-4 |

Table 3: Measured heights (m), diameters (m) and areas (m2) in Venturi apparatus.

Time to fill 500 millilitres of water directly from the Venturi apparatus:

|  |
| --- |
| Time (s) |
| 8.40  8.25  7.96 |

Table 4: Measured time (s) to fill 500ml of water into a beaker from Venturi apparatus.

Analysis

Experiment One – Continuity and Bernoulli Equation

The velocity of the water at the surface of the beaker (V1) was found by plotting the water height in the beaker against time on a graph. The gradient of the line of best fit was therefore the velocity of the water at the surface of the beaker. The uncertainty bars went both vertically and horizontally on the graph. This was because the uncertainty of time and the water height were relatively small compared to the scale that the graph adopted.

Figure 3: Graph for finding the velocity of water through the hole using the gradient.

Velocity using continuity equation

The velocity of the water through the hole in the beaker was calculated using this velocity. This was achieved by using the continuity equation **(3)** which was manipulated to find V2 (velocity of water thought the hole). The area of the beaker and hole was calculated using the formula:

|  |  |  |
| --- | --- | --- |
|  |  | (14) |

Thus V2 was able to be calculated using these areas.

|  |  |  |
| --- | --- | --- |
|  |  | (15) |

**Uncertainties:**

The percentage uncertainty for the area of both the beaker and hole were determined by the uncertainty of the radius as Pi is a constant and has no uncertainty.

Since the radius was squared, the percentage uncertainty of the radius must be doubled.

The same method was used to find the uncertainty of the area of the hole.

The uncertainty of the velocity of the water at the surface of the beaker was created by the uncertainty of the time and uncertainty of the height of the water level in the beaker.

The percentage uncertainty for time was calculated by halving the range of trying to make the timer read exactly one second as stated earlier in the report.

To find the total uncertainty of the continuity equation, the percentage uncertainties are added together.

Thus the velocity of the water leaving the beaker through the hole using the continuity equation was calculated to be:

Velocity using Bernoulli’s Equation

The velocity of the water leaving the hole was also calculated using the rearranged Bernoulli equation (13).

**Uncertainty:**

The only value with an associated uncertainty in this equation was the surface height. The surface height was 0.100 ± 0.001. Since this value was inside the square root function, the brute force method was used to calculate the uncertainty of the calculated velocity.

Thus the velocity of the water leaving the hole using Bernoulli’s rearranged equation was calculated to be:

Experiment 2 – Venturi Effect

The second experiment required the average flow rate of the water through the Venturi apparatus to be calculated. The change in volume and time were recorded in the results section of this report. The flow rate was averaged and that was the final flowrate which was used to find the velocity of the water which was needed in the continuity equation.

|  |  |  |
| --- | --- | --- |
|  |  | (16) |

The change in volume was one litre and the change in time was the time required to fill 500 millilitres of water in a beaker from the Venturi apparatus multiplied by two.

To convert the average flow rate from litres per second into metres cubed per second:

Velocity of water at each position using the continuity equation

The velocity of the water at each position in the Venturi apparatus was calculated using equation **(3)**. It was manipulated and then the equation was used for all the holes. Here is a sample calculation using the results obtained from the first position:

|  |  |  |
| --- | --- | --- |
|  |  | (17) |

This was repeated for each position and the velocities of the water at each hole were recorded in the table below.

|  |  |
| --- | --- |
| Position | Velocity using  Continuity (ms-1) |
| 1  2  3  4  5  6  7 | 0.368  0.420  0.458  0.500  0.554  0.610  0.678 |

Table 5: Measured velocity (ms-1) of water at each point in the Venturi apparatus using the continuity equation.

Velocity of water at each position using Bernoulli’s equation.

In order to find the velocity of the water at each nth position, equation (13) was used.

|  |  |  |
| --- | --- | --- |
|  |  |  |

Below is a sample calculation using the data from position one.

This was repeated for every position. The calculated velocities were recorded in the table below.

|  |  |
| --- | --- |
| Position | Velocity using Bernoulli’s (ms-1) |
| 1  2  3  4  5  6  7 | 0.505  0.423  0.485  0.542  0.672  0.729  0.863 |

Table 6: Measured velocity of water (ms-1) in each hole in the Venturi apparatus using Bernoulli's equation.

Percentage difference between continuity and Bernoulli’s equation

The percentage difference of the velocity of water at each position using the continuity equation and Bernoulli’s equation was calculated by using the equation:

|  |  |  |
| --- | --- | --- |
|  | (Calculator Soup, 2017) | (18) |

For example, the difference in the values for the velocity of water at position one was calculated as:

This was repeated for every position. The percentages were noted in the table below.

|  |  |
| --- | --- |
| Position | % Difference |
| 1  2  3  4  5  6  7 | 31  0.7  5.7  8.1  19  18  24 |

Table 7: Percentage difference in the resulting velocities using the continuity equation and Bernoulli's equation.

Discussion

Experiment One – Bernoulli and Continuity Equation

Using the continuity equation, the velocity of the water through the hole in the beaker was determined to be (0.990 ± 0.273) ms-1. The velocity of the water through the hole in the beaker was calculated to be (1.400 ± 0.007) ms-1 using Bernoulli’s equation. While these results did not agree with each other even after their uncertainties were taken into account, they were pretty similar. The reason that they did not agree could have been due to the amount of variables in the continuity equation compared to Bernoulli’s equation. The continuity equation used time, height and velocity variables which introduced more chance of errors in the experiment. However Bernoulli’s equation only deals with the height of the water in the beaker. Therefore Bernoulli’s equation should be the more accurate equation to use. This was also the reason why the uncertainty was much larger for the continuity equation compared to that of Bernoulli’s equation.

In reality, neither of these equations are accurate as the water was not flowing with ideal flow. The flow through the hole was not laminar and there were friction forces acting against the flow of the water.

The theory used in this experiment can be seen applied everywhere around the world. For example, instead of a beaker, what if it was an eighty metre tall water reservoir on top of a hill just like what is the case in some cities? The velocity of the water leaving the water reservoir through a spout at the end could be calculated using equation (7). Here is the calculation:

However, when someone opens their tap to fill a glass of water, the water does not flow out at 40ms-1. This is due to the flow of the water in the pipes not being ideal. The water flow is not streamline or laminar. Therefore there would be a loss of speed as some of the kinetic energy of the water is turned into heat energy due to friction.

Experiment Two – Venturi Effect

The velocity of the water in the Venturi tube at each position using the continuity equation was (0.368, 0.420, 0.458, 0.500, 0.554, 0.610 and 0.678) ms-1. However, the calculated velocities using Bernoulli’s equation were (0.505, 0.423, 0.485, 0.542, 0.672, 0.729 and 0.863) ms-1. The resulting difference between the calculated velocities were (31, 0.7, 5.7, 8.1, 19, 18 and 24) % respectively. The reason that these percentages ranged from 0.7% to 31% could have been due to the experiment conditions. Since the Venturi tubed was connected directly to a tap, if anyone else turned on their tap, the pressure of the water in the tap would decrease to accommodate the extra flow required. Therefore this made the flow of water in the Venturi apparatus fluctuate which would have changed the heights measured in the manometer tubes. Therefore, the water pressure would sometimes change between recording the data for the static height and dynamic height. Using both equations, the velocity of the water tends to increase as the Pitot tube moves from one hole to the next. However, the velocity calculated with Bernoulli’s equation at position one did not follow this trend. This was once again could have been due to the fluctuating water pressure in the pipe outlet.

But why does the velocity of the water increase at each point of the Venturi apparatus? When the area of the tube decreases i.e. the bigger the hole number in the Venturi tube, the narrower flow creates a vacuum in the flow. As a result, the pressure at that point decreases and the kinetic energy of the fluid increases. This means that the velocity of the water must increase as well. (Tech Faq, 2017).

Using these equations provided a theoretical velocity of the water. However in reality, this was not the true velocity of the water as the water could never have been flowing with ideal flow. It would be physically impossible to make sure water flowed laminar and it would also be impossible to not have friction working against the flow of water. Therefore, due to the physical limitations on the experiment, the velocities obtained were theoretical and would not be the actual velocities of water in these two experiments.

Conclusion

The aim of this experiment was to use the continuity equation and Bernoulli’s equation in the hole in a beaker experiment and Venturi tube to improve understanding of fluid dynamics. The velocity of the water through the hole in the beaker using the continuity equation was calculated to be (0.990 ± 0.273) ms-1. However the velocity through the hole was calculated to be (1.400 ± 0.007) ms-1. The difference in the velocities could have been due to the continuity equation taking more variables into account i.e. the measured height of the water and the time required to drain the beaker whereas Bernoulli’s equation only took the measured height of the water into account. The velocity of water at each hole in the Venturi tube using the continuity equation (0.368, 0.420, 0.458, 0.500, 0.554, 0.610 and 0.678) ms-1. The velocity of water at each hole in the Venturi tube was calculated to be (0.505, 0.423, 0.485, 0.542, 0.672, 0.729 and 0.863) ms-1 using Bernoulli’s equation. There was a percentage difference of (31, 0.7, 5.7, 8.1, 19, 18 and 24) % between the results using each equation. The varied percentage difference could have been due to the water pressure fluctuating during recording of the data. However, these equations only calculated theoretical velocities as they assumed laminar flow of the water and that friction did not act against the flow of water.

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